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This study compared the performances of college mathematics students on mathematics problems stated in three different contexts: in arithmetical form, in the form of algebraic expressions to be simplified, and in the form of practical situations stated in words. Both single-step and multi-step problems were used. For all three contexts, there were more errors made on the multi-step problems. However, in contrast to results reported in the literature for precollege students, there was no significant difference in the students' performances for the three types of contexts. The study also investigated the perception of the difficulty of solving word problems as compared to other types of problems in mathematics. It was found that students tended to perceive word problems as more difficult. While perception of difficulty had no overall relationship to performance in solving word problems, a significant perception of difficulty x single- versus multi-step interaction was uncovered. This interaction resulted largely from poorer performance in solving multi-step problems by students who perceived word problems as more difficult. (Author)

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College Mathematics Students' Reactions to Word Problems: Perceptions and Performance

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ABSTRACT

This study compared the performances of college mathematics students on mathematics problems stated in three different contexts: in arithmetical form, in the form of algebraic expressions to be simplified, and in the form of practical situations stated in words. Both single-step and multi-step problems were used. For all three contexts, there were more errors made on the multi-step problems. However, in contrast to results reported in the literature for pre-college students, there was no significant difference in the students' performances for the three types of contexts.

The study also investigated the perception of the difficulty of solving word problems as compared to other types of problems in mathematics. It was found that students tended to perceive word problems as more difficult. While perception of difficulty had no overall relationship to performance in solving word problems, a significant perception of difficulty x single versus multi-step interaction was uncovered. This interaction resulted largely from a poorer performance in solving multi-step than single-step problems by the students who perceived word problems as more difficult.

One of the important skills that we try to impart to our students is the ability to solve mathematics problems stated in verbal form. A series of national mathematics assessments documents the difficulty students have in solving word problems. For example, in spite of the fact that 74% of the 17-year-old students surveyed in The Second National Assessment of Educational Progress in Mathematics (Education Commission of the States, 1979) correctly multiplied 2/3 x 2/5, only 21% could solve a word problem similar to the following:

> Jane lives 2/3 mile from school. When she has walked 2/5 of the way, how far has she walked?

The assessment also indicated that junior high and secondary school students had considerable difficulty with multi-step word problems, even when the calculations involved were well within the range of the students' computational skills Carpenter, Corbitt, Kepner, Lindquist, and Reys, 1981).

A subsequent National Assessment of Educational Progress in Mathematics (Educational Commission of the States, 1983) indicated little change in the ability of students to solve word problems. The only apparent gain reported was a 2% gain by 13-year-old students in solving nonroutine word problems (Carpenter, Lindquist, Matthews, and Silver, 1983).

While the most recent National Assessment of Educational Progress (Educational Testing Service, 1986) indicated that 7th and 11th grade students were about as successful in solving simple one-step word problems dealing with percents as they were in performing similar calculations, the report once again highlighted the difficulties students had in solving multi-step

word problems (Brown, Carpenter, Kouba, Lindquist, Silver, and Swafford, 1988). Students tended to view multi-step problems simplistically, often performing only one arithmetical operation, when several were required to solve the problem correctly.

Teaching word problems has presented a persistent challenge for educators at the junior high and secondary levels. A variety of approaches to address this problem have been suggested. These include writing exercises (Johnson, 1983; Hendricks and Sisson, 1980; Brink, 1982; and Denmark, 1983), verbalization (Bonjoh, 1982), focus on the analysis of the language of the problem (Robbins and Hauge, 1982), using representation diagrams and basing instruction on cognitive analyses of students' prior knowledge (Lewis, 1989).

Difficulties presented to students by word problems may not end with high school. Teaching word problems may also present a problem for math instruction in the first two years of college as well. A report published by The National Center for Educational Statistics (1985) indicated that a very large number of students, one-fourth of the 1983-1984 college freshmen had signed up for remedial math, which, of course, includes word problems. In addition, many colleges and universities offer other courses in math as well as in other disciplines which include problems stated in verbal form.

As research suggests, the teaching of word problems at the pre-college level requires special emphasis, we wonder if this need still persists at the college level. Should teachers at the college level be sensitized to special difficulties posed by word problems?

There has been relatively little research carried out on solving word problems at the college level. While studies have

investigated different techniques for solving word problems at this level (e.g., Whimbey and Lochhead, 1980; Mett, 1985 and Lewis, 1989) there is little data available which address the basic question as to whether word problems pose a special difficulty for college students. One study carried out on the problem (Clement, Lochhead, and Monk, 1981) found that students had less difficulty solving proportion problems formulated algebraically than in algebraically formulating proportion problems stated in words (Clement et al., 1981). This seems consistent with the added difficulties found by pre-college students in solving certain word problems.

The present study was carried out to provide further data bearing on this question. The plan of the study was to compare the performances of college math students on mathematics problems stated in three different contexts: in the form of practical situations stated in words, in the form of algebraic expressions to be simplfied, and in arithmetical form. In view of the findings reported for multi-step word problems in the assessments (e.g, The Fourth Mathematics Assessment of the National Assessment of Educational Progress, 1986), both single-step and multi-step problems were included in each of these contexts.

PROCEDURE

This study was carried out at an open admissions urban university. The subjects were all the students (36 males and 53 females) enrolled in three sections of a freshmen-level

survey course in mathematics designed for non-mathematics majors. In regard to the reading level of the subjects, the students had been tested for reading ability by the University using a standardized test, the Reading Comprehension Test (College Entrance Examination Board, 1977). Based on these test scores, 32% of the subjects completed a course in remedial reading.

The design of the study was as follows. Initially a brief questionnaire was administered to assess the students' perceptions of the difficulty of solving word problems, as well as their prior education in two areas, math courses and verbally-oriented courses (e.g., reading, English composition, and literature).

The question that was used to assess perceptions of difficulty in solving word problems was as follows:

> Compared with other math problems that you have had in courses you have taken, did you find word problems
>
> ( ) Almost impossible
> ( ) More difficult
> ( ) Of about the same difficulty
> ( ) Less difficult.

The questions about prior training in math and verbally-related courses asked the students to check the courses they had taken in both high school and college in these areas, and to indicate the grades they had received. The number of courses taken in each of these areas and the grade-point averages achieved were computed for each student for both high school and college.

One week after the questionnaire data had been collected,

an experiment was carried out to assess the students' performance in solving twelve math problems, each stated in the three different contexts. The level of difficulty of the problems chosen was roughly equivalent to those encountered on the Applied Arithmetic Placement Test (College Entrance Examination Board, 1972).

The test problems were administered in the following way. Three formats of math problems were given out one week apart. Format A presented twelve math problems stated in words. Format B presented the twelve problems as algebraic expressions, containing both numbers and letters, to be simplified, while Format C presented the problems in arithmetical form, using only numbers.

The formats were rotated among the three groups of subjects and the problems were presented in a different order on each of the tests.

Calculators were not allowed. The students were given no information about their performance on any of the tests, until they had completed all of the tests. After they had completed one of the formats, they were not told that they would be asked to do the other formats, until the next format was administered.

Each of the tests consisted of six one-step problems and six multi-step problems. A typical example of each type problem on each type format follows:

One-step Problem:

Format A. The gas tank of an automobile holds 21 $\frac{1}{2}$ gallons. How many gallons are in the tank if it is $\frac{3}{4}$ full?

Format B. Simplify: $(22\ \frac{1}{2})\ \frac{g}{t} \times \frac{2t}{9} =$

Format C. Simplify: $24\ \frac{1}{2} \times \frac{2}{9} =$

Multi-step Problem:

Format A. If it costs \$0.50 for the first 10 words and \$0.02 per each additional word, how much does it cost to send a 32-word telegram?

Format B. Simplify: $0.35d + \frac{0.03d}{w}(53w - 22w) =$

Format C. Simplify: $0.75 + 0.03(51 - 20) =$

To prevent a possible memory effect, different numbers were used for presenting the related problems in each of the three contexts. To ensure that the computations used in the three related problems on each of the three formats were of equal difficulty, the numbers used in each of the related problems were pretested in the following way. The numbers were extracted from each problem in the algebraic and word problem formats and were presented as arithmetic problems as in Format C. These arithmetic problems together with the arithmetic problems on Format C were arranged in random order and given as an arithmetic test to all the students (17 males, 21 females) in two other sections of the same freshmen level survey course. Performance by the students on each of the sets of related arithmetic problems was analyzed on an item-by-item

basis within each set. The arithmetic difficulties for the three arithmetic problems within a set were extremely comparable: the median range of the percent of correct responses within the sets of related problems was less than one percent.

RESULTS

Perception of Difficulty of Solving Word Problems

Aiken (1970) suggests that a student's attitude toward problem solving may be very different from his/her attitude toward computation. In line with this observation, our first question was whether college students had a tendency to perceive word problems as more difficult to solve than other types of math problems. The students' responses to the question asking about the difficulty in solving word problems are presented in Table 1. A Chi-square test indicated that the responses were not equally distributed, $\chi^2(3, n = 89) = 20.52$, $p < .01$.

(Table 1 about here)

One may observe that almost three times as many students responded "more difficult" than "less difficult." To confirm this observation, we asked a second sample of thirty-five students (14 males, 21 females) taking a survey course in college mathematics the same question concerning their perception of the difficulty of solving word problems in mathematics, this time restricting the choices for their responses to "more difficult," "less difficult" and "of about the same difficulty." Seventy-four percent of these students responded "more difficult"; six percent, "less difficult"; and twenty percent, "of about the same difficulty."

Performance on Solving the Test Problems

Having noted a tendency for students to believe word problems were more difficult than other types of problems in mathematics, let us now turn to the actual performance of our primary sample of students. Was there, in fact, a difference between their performance in solving the word problems and their performance in solving the other types of problems?

Preliminary ANOVAs were run to establish that the three sections of students in our sample were homogeneous in terms of their scores on the test problems. An initial ANOVA using the total number of items each student answered correctly on all three tests (word problems, algebraic expressions to be simplified, and problems stated in arithmetical form, using only numbers) as the dependent measure of performance revealed that the scores were extremely similar, ($F < 1$). Similarly, the sections were found to be homogeneous in terms of their performance on each of the three individual formats (word, algebra, arithmetic). The three sections were therefore combined into a single sample for the major analyses.

Scores on the three measures, the number of correct answers on each type of format tended to be substantially intercorrelated (arithmetic versus algebra, .63; arithmetic versus word problems, .78; algebra versus word problems, .55).

Table 2 presents the means and standard deviations of correct answers on the single and multi-step problems for each of the three formats (math problems stated in words, math problems stated as algebraic expressions to be simplified, and math

problems stated in arithmetical form using only numbers).

(Table 2 about here)

A 2 x 3 ANOVA with repeated measurements indicated (a) Student performance on the three types of formats was not significantly different, ($F < 1$); (b) Student performance on the multi-step problems was significantly poorer than on the single-step problems, $F(1,88) = 90.77$, $p < .01$; (c) The interaction of type of format x single-step vs multi-step was not significant, ($F < 1$).

One might wonder whether the students who perceived word problems as more difficult performed less well on the word problems than those who believed word problems presented no added difficulty. To study this question, the students were dichotomized into two groups: Those who perceived word problems as relatively more difficult to solve ("almost impossible" and "more difficult") and those who did not ("of about the same level of difficulty" and "less difficult"). In a 2 x 2 ANOVA these groups were compared in terms of their performance on single-step and multi-step word problems.

(Table 3 about here)

There was no main effect for perception of word problem difficulty on solving word problems $F(1,87) = 2.14$, $p > .05$. The main effect for single versus multi-step was significant, $F(1,87) = 151$, $p < .001$, and there was a perception x single versus multi-step interaction, $F(1,87) = 32$, $p < .001$. Post hoc tests for simple effects indicated that both groups of students (those who perceived word problems as more difficult and

those who did not perceive word problems as more difficult) performed better on the single-step word problems than on the multi-step word problems, $F(1, 87) = 176$, $p < .001$ and $F(1,87) = 8.64$, $p < .005$. However, it was clear that the magnitudes of these differences in performance were not similar. The difference in performance on single and multi-step problems appeared to be much greater for the students who perceived word problems as more difficult. A t test comparing the two sets of difference scores (score on single-step minus score on multi-step word problems) for the two groups of students was significant, $t(87) = 8.0$, $p < .001$).

In a final analysis, we found that the background variables in math and verbally-related education (both the number of courses taken and the grades attained in either area) were not related to student performance on the word problem test.

DISCUSSION

The study examined the performance of university students taking a freshmen-level survey course in college mathematics on word problems in mathematics. We found that students tended to perceive word problems as more difficult to solve than other types of problems. However, we also found that the students' performance on the word problem test was no different from their performance on the algebra and arithmetic tests. The picture presented by this finding stands in contrast to the results reported in some of the national assessments for pre-college students where performance on word problems was markedly poorer

than performance on other types of mathematics problems.

In generalizing from the present findings, one should keep in mind that the procedure employed a range of difficulty in the word problems used which was roughly equivalent to that of The Applied Arithmetic Placement Test of The College Entrance Examination Board. These word problems both in vocabulary level and context were well within the realm of the students' experiences. It may be that the students' familiarity with the kinds of situations in these problems reduced the difficulties posed by the word problem format. It is possible that the use of word problems which involve terms less familiar to the students (e.g., science and technology) would produce different results. Such a possibility merits further investigation.

Another central finding of the study was that students had more difficulty solving multi-step problems than they experienced solving single-step problems. This finding seems consistent with earlier findings reported in the national assessments for junior high and high school students who were more successful solving routine problems than problems requiring more analysis (Carpenter et al., 1983 and Brown et al., 1988). In interpreting our results, one should keep in mind that there is a greater chance of making arithmetical errors in multi-step problems than in single-step problems, because of the greater number of calculations involved. Beyond that we need to know more about students' reactions to multi-step problems both in terms of the students' cognitive

processes in approaching such problems and in terms of possible confusion and anxiety that might be aroused.

An interesting finding was the perception of difficulty x single vs multi-step interaction for solving word problems. The interaction resulted largely from a poorer performance in solving multi-step than single-step problems by the students who perceived word problems as more difficult. These data suggest that multi-step word problems may present a special problem to students who believe word problems are difficult.

Implications for Classroom Teaching

The findings from this study have implications for classroom teaching. First, word problems tend to be perceived by students as particularly difficult when compared with problems that only involve arithmetic or algebraic operations. Presentation of word problems in the classroom may arouse negative expectations in some students such as "It's too difficult" or "I can't solve it." Students approaching the problems this way may be easily discouraged, increasing the chances of failure.

In attempting to develop more positive attitudes toward the solution of word problems, the teacher might want to try experimenting with strategies that maximize the probability of success experiences for the student. One such strategy would be to begin with problems that are easy and describe situations in which the students have a great deal of direct experience. When the students experience success with such problems, the teacher could then apply the principles learned to more abstract problems such as one might encounter in the sciences.

This procedure for teaching word problem solving skills by beginning with problems that are easy and well within the experience of the students is, of course, consistent with our observation that the students in the current study had no added difficulty in solving word problems that described practical, familiar situations.

Another major implication of our findings for classroom instruction concerns teaching single-step vs multi-step problems. The difficulties students experience with multi-step problems at all educational levels suggests the need to place additional emphasis on analyzing techniques for solving mathematics problems that require more than the application of a single operation. Such instruction might stress the following: (a) What information is given in the problem, (b) What is to be found, (c) The relationship(s) of what is given to what is to be found, (d) The operations that are involved, (e) The order in which the operations are to be performed, (f) The information that is obtained by performing each of the operations, and (g) Why this information is essential to the solution of the problem.

Certain procedures have been suggested to make students more aware of their thought processes while solving multi-step problems. Examples include Hayes' and Flower's suggestion of having students speak their thoughts while writing (1978), verbalization (Whimbey and Lochhead, 1980), directed writing assignments (Mett, 1985) and representation diagrams (Lewis, 1989). The latter procedures are now being explored in certain courses at the college level.

In teaching students to solve multi-step problems in mathematics, the teacher might find it helpful to experiment with different learning techniques that may involve various modalities. These techniques might include invoking the user friendly language capability of a computer; having students outline the steps for solving the problem; drawing diagrams such as flowcharts; and encouraging discussions in which students present and analyze multi-step problems. Some students may respond better to one approach rather than another, and the use of different modalities might have the effect of reenforcing the learning process involved in multi-step problem solving.

Table 1

Student's Perceptions of the Difficulty of Solving Word Problems (n = 89)

Student Perceptions	Percent of Students
Almost impossible	8
More difficult	48
About the same difficulty	26
Less difficult	18

Table 2

Means and Standard Deviations of Correct Answers on Single-step and Multi-step Problems on Each of The Three Types of Tests (n = 89)

Type of test	$\bar{X}$	s
Arithmetic		
Single-step	4.7	1.4
Multi-step	2.9	1.7
Algebra		
Single-step	4.8	1.1
Multi-step	2.6	1.5
Word		
Single-step	4.9	1.2
Multi-step	3.0	1.5

Table 3

Means and Standard Deviations of Correct Answers on Single and Multi-step Word Problems by Students Who Perceived Word Problems as More Difficult and by Students Who Perceived Word Problems as No More Difficult

	Students Who Perceived Word Problems as More Difficult (n = 50)		Students Who Perceived Word Problems as No More Difficult (n = 39)	
Type of Test	$\bar{X}$	s	$\bar{X}$	s
Word Problems				
Single-step	5.05	.98	4.44	1.22
Multi-step	2.44	1.31	3.69	1.23

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